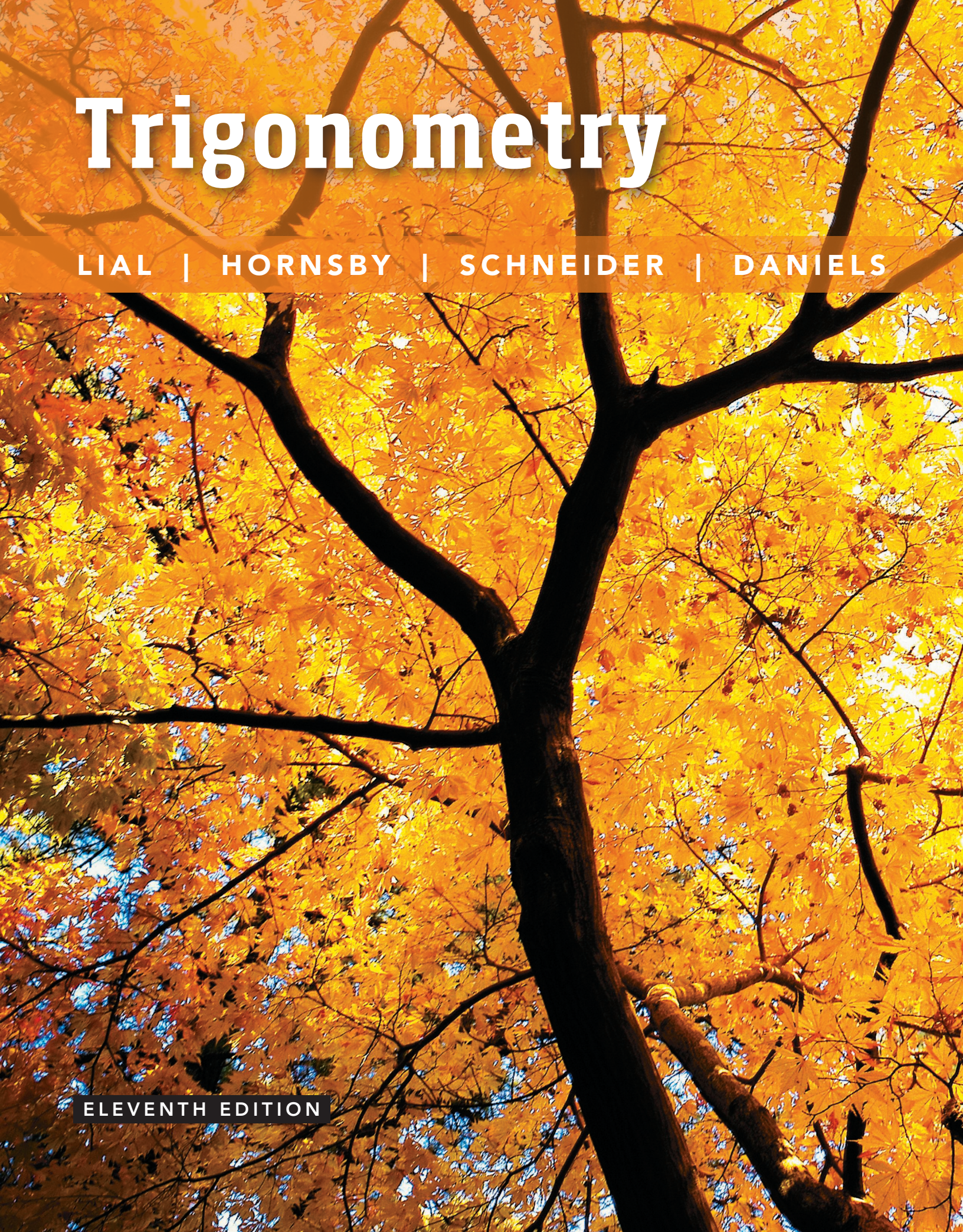
Trigonometry

LIAL | HORNSBY | SCHNEIDER | DANIELS



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These MyMathLab resources will help you think visually and connect the concepts.

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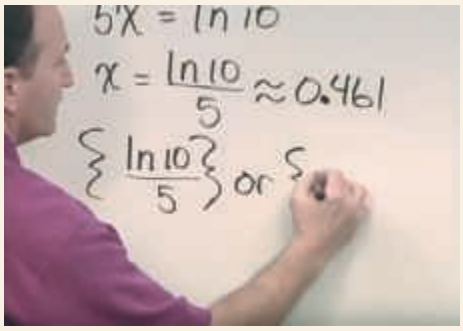
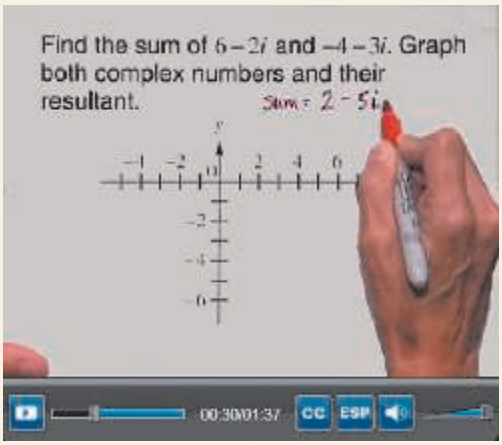
Updated videos are available to support students outside of the classroom and cover all topics in the text. Quick Review videos cover key definitions and procedures. Example Solution videos offer a detailed solution process for every example in the textbook.





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Assignable in MyMathLab, Example Solution Videos present the detailed solution process for every example in the text. Additional Quick Reviews cover definitions and procedures for each section. Assessment exercises check conceptual understanding of the mathematics.

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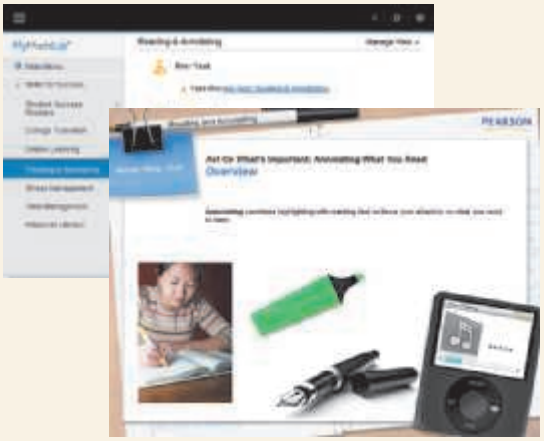

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Trigonometry

ELEVENTH EDITION

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PEARSON

Boston Columbus Indianapolis New York San Francisco
Amsterdam Cape Town Dubai London Madrid Milan Munich Paris Montréal Toronto
Delhi Mexico City São Paulo Sydney Hong Kong Seoul Singapore Taipei Tokyo

To Butch, Peggy, Natalie, and Alexis–and in memory of Mark

E.J.H.

To Coach Lonnie Myers–thank you for your leadership on and off the court.

C.J.D.

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Contents

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Preface

WELCOME TO THE 11TH EDITION

In the eleventh edition of *Trigonometry,* we continue our ongoing commitment to providing the best possible text to help instructors teach and students succeed. In this edition, we have remained true to the pedagogical style of the past while staying focused on the needs of today's students. Support for all classroom types (traditional, hybrid, and online) may be found in this classic text and its supplements backed by the power of Pearson's MyMathLab.

In this edition, we have drawn upon the extensive teaching experience of the Lial team, with special consideration given to reviewer suggestions. General updates include enhanced readability with improved layout of examples, better use of color in displays, and language written with students in mind. All calculator screenshots have been updated and now provide color displays to enhance students' conceptual understanding. Each homework section now begins with a group of *Concept Preview* exercises, assignable in MyMathLab, which may be used to ensure students' understanding of vocabulary and basic concepts prior to beginning the regular homework exercises.

Further enhancements include numerous current data examples and exercises that have been updated to reflect current information. Additional real-life exercises have been included to pique student interest; answers to writing exercises have been provided; better consistency has been achieved between the directions that introduce examples and those that introduce the corresponding exercises; and better guidance for rounding of answers has been provided in the exercise sets.

The Lial team believes this to be our best *Trigonometry* edition yet, and we sincerely hope that you enjoy using it as much as we have enjoyed writing it. Additional textbooks in this series are as follows:

College Algebra, Twelfth Edition
College Algebra & Trigonometry, Sixth Edition
Precalculus, Sixth Edition

HIGHLIGHTS OF NEW CONTENT

- Discussion of the Pythagorean theorem and the distance formula has been moved from an appendix to **Chapter 1.**
- In **Chapter 2,** the two sections devoted to applications of right triangles now begin with short historical vignettes, to provide motivation and illustrate how trigonometry developed as a tool for astronomers.
- The example solutions of applications of angular speed in **Chapter 3** have been rewritten to illustrate the use of unit fractions.
- In **Chapter 4,** we have included new applications of periodic functions. They involve modeling monthly temperatures of regions in the southern hemisphere and fractional part of the moon illuminated for each day of a particular month. The example of addition of ordinates in **Section 4.4** has been rewritten, and a new example of analysis of damped oscillatory motion has been included in **Section 4.5.**
- **Chapter 5** now presents a derivation of the product-to-sum identity for the product $\sin A \cos B$.
- In **Chapter 6,** we include several new screens of periodic function graphs that differ in appearance from typical ones. They pertain to the music phenomena of pressure of a plucked spring, beats, and upper harmonics.

- The two sections in **Chapter 7** on vectors have been reorganized but still cover the same material as in the previous edition. **Section 7.4** now introduces geometrically defined vectors and applications, and **Section 7.5** follows with algebraically defined vectors and the dot product.
- In **Chapter 8,** the examples in **Section 8.1** have been reordered for a better flow with respect to solving quadratic equations with complex solutions.
- For visual learners, numbered **Figure** and **Example** references within the text are set using the same typeface as the figure number itself and bold print for the example. This makes it easier for the students to identify and connect them. We also have increased our use of a "drop down" style, when appropriate, to distinguish between simplifying expressions and solving equations, and we have added many more explanatory side comments. Guided Visualizations, with accompanying exercises and explorations, are now available and assignable in MyMathLab.
- *Trigonometry* is widely recognized for the quality of its exercises. In the eleventh edition, nearly 500 are new or modified, and many present updated real-life data. Furthermore, the MyMathLab course has expanded coverage of all exercise types appearing in the exercise sets, as well as the mid-chapter Quizzes and Summary Exercises.

FEATURES OF THIS TEXT

SUPPORT FOR LEARNING CONCEPTS

We provide a variety of features to support students' learning of the essential topics of trigonometry. Explanations that are written in understandable terms, figures and graphs that illustrate examples and concepts, graphing technology that supports and enhances algebraic manipulations, and real-life applications that enrich the topics with meaning all provide opportunities for students to deepen their understanding of mathematics. These features help students make mathematical connections and expand their own knowledge base.

- **Examples** Numbered examples that illustrate the techniques for working exercises are found in every section. We use traditional explanations, side comments, and pointers to describe the steps taken—and to warn students about common pitfalls. Some examples provide additional graphing calculator solutions, although these can be omitted if desired.
- **Now Try Exercises** Following each numbered example, the student is directed to try a corresponding odd-numbered exercise (or exercises). This feature allows for quick feedback to determine whether the student has understood the principles illustrated in the example.
- **Real-Life Applications** We have included hundreds of real-life applications, many with data updated from the previous edition. They come from fields such as sports, biology, astronomy, geology, music, and environmental studies.
- **Function Boxes** Special function boxes offer a comprehensive, visual introduction to each type of trigonometric function and also serve as an excellent resource for reference and review. Each function box includes a table of values, traditional and calculator-generated graphs, the domain, the range, and other special information about the function. These boxes are assignable in MyMathLab.
- **Figures and Photos** Today's students are more visually oriented than ever before, and we have updated the figures and photos in this edition to

promote visual appeal. Guided Visualizations with accompanying exercises and explorations are now available and assignable in MyMathLab.

- **Use of Graphing Technology** We have integrated the use of graphing calculators where appropriate, although ***this technology is completely optional and can be omitted without loss of continuity.*** We continue to stress that graphing calculators support understanding but that students must first master the underlying mathematical concepts. Exercises that require the use of a graphing calculator are marked with the icon .
- **Cautions and Notes** Text that is marked **CAUTION** warns students of common errors, and **NOTE** comments point out explanations that should receive particular attention.
- **Looking Ahead to Calculus** These margin notes offer glimpses of how the topics currently being studied are used in calculus.

SUPPORT FOR PRACTICING CONCEPTS

This text offers a wide variety of exercises to help students master trigonometry. The extensive exercise sets provide ample opportunity for practice, and the exercise problems generally increase in difficulty so that students at every level of understanding are challenged. The variety of exercise types promotes understanding of the concepts and reduces the need for rote memorization.

- **NEW Concept Preview** Each exercise set now begins with a group of **CONCEPT PREVIEW** exercises designed to promote understanding of vocabulary and basic concepts of each section. These new exercises are assignable in MyMathLab and will provide support especially for hybrid, online, and flipped courses.
- **Exercise Sets** In addition to traditional drill exercises, this text includes writing exercises, optional graphing calculator problems , and multiple-choice, matching, true/false, and completion exercises. ***Concept Check*** exercises focus on conceptual thinking. ***Connecting Graphs with Equations*** exercises challenge students to write equations that correspond to given graphs.
- **Relating Concepts Exercises** Appearing at the end of selected exercise sets, these groups of exercises are designed so that students who work them in numerical order will follow a line of reasoning that leads to an understanding of how various topics and concepts are related. All answers to these exercises appear in the student answer section, and these exercises are assignable in MyMathLab.
- **Complete Solutions to Selected Exercises** Complete solutions to all exercises marked are available in the eText. These are often exercises that extend the skills and concepts presented in the numbered examples.

SUPPORT FOR REVIEW AND TEST PREP

Ample opportunities for review are found within the chapters and at the ends of chapters. Quizzes that are interspersed within chapters provide a quick assessment of students' understanding of the material presented up to that point in the chapter. Chapter "Test Preps" provide comprehensive study aids to help students prepare for tests.

- **Quizzes** Students can periodically check their progress with in-chapter quizzes that appear in all chapters. All answers, with corresponding section references, appear in the student answer section. These quizzes are assignable in MyMathLab.

- **Summary Exercises** These sets of in-chapter exercises give students the all-important opportunity to work *mixed* review exercises, requiring them to synthesize concepts and select appropriate solution methods.
- **End-of-Chapter Test Prep** Following the final numbered section in each chapter, the Test Prep provides a list of **Key Terms,** a list of **New Symbols** (if applicable), and a two-column **Quick Review** that includes a section-by-section summary of concepts and examples. This feature concludes with a comprehensive set of **Review Exercises** and a **Chapter Test.** The Test Prep, Review Exercises, and Chapter Test are assignable in MyMathLab. Additional Cumulative Review homework assignments are available in MyMathLab, following every chapter.

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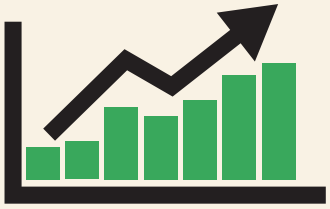

MyMathLab is the world's leading online resource for teaching and learning mathematics. MyMathLab helps students and instructors improve results, and it provides engaging experiences and personalized learning for each student so learning can happen in any environment. Plus, it offers flexible and time-saving course management features to allow instructors to easily manage their classes while remaining in complete control, regardless of course format.

Personalized Support for Students

- MyMathLab comes with many learning resources–eText, animations, videos, and more–all designed to support your students as they progress through their course.
- The Adaptive Study Plan acts as a personal tutor, updating in real time based on student performance to provide personalized recommendations on what to work on next. With the new Companion Study Plan assignments, instructors can now assign the Study Plan as a prerequisite to a test or quiz, helping to guide students through concepts they need to master.
- Personalized Homework enables instructors to create homework assignments tailored to each student's specific needs and focused on the topics they have not yet mastered.

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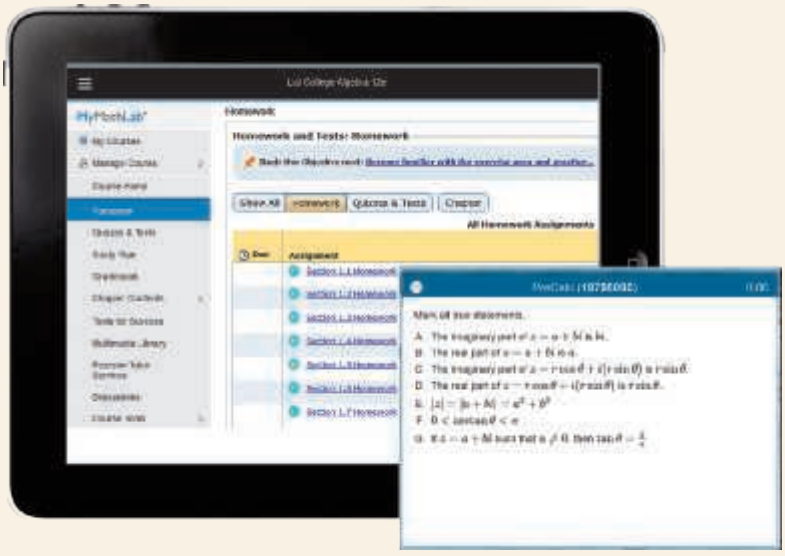
Resources for Success

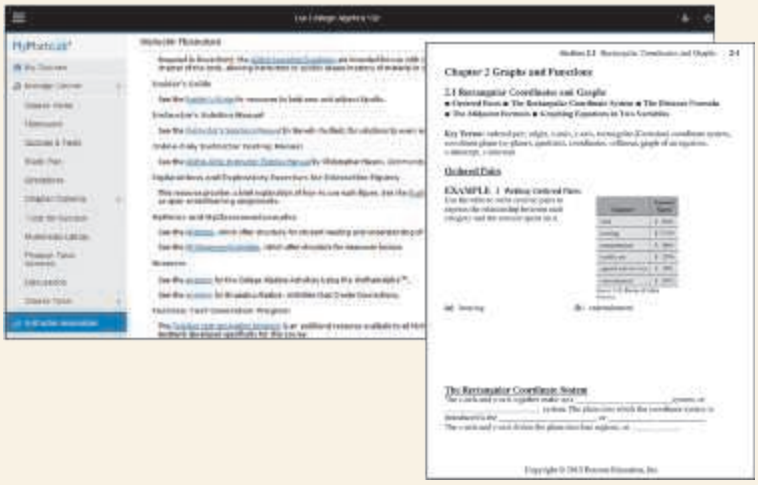
MyMathLab® Online Course for *Trigonometry* by Lial, Hornsby, Schneider, and Daniels

MyMathLab delivers proven results in helping individual students succeed. The authors Lial, Hornsby, Schneider, and Daniels have developed specific content in MyMathLab to give students the practice they need to develop a conceptual understanding of Trigonometry and the analytical skills necessary for success in mathematics. The MyMathLab features described here support Trigonometry students in a variety of classroom formats (traditional, hybrid, and online).

Concept Preview Exercises

Exercise sets now begin with a group of Concept Preview Exercises, assignable in MyMathLab and also available in Learning Catalytics. These may be used to ensure that students understand the related vocabulary and basic concepts before beginning the regular homework problems. Learning Catalytics is a "bring your own device" system of prebuilt questions designed to enhance student engagement and facilitate assessment.





MyNotes and MyClassroomExamples

MyNotes provide a note-taking structure for students to use while they read the text or watch the MyMathLab videos. MyClassroom Examples offer structure for notes taken during lecture and are for use with the Classroom Examples found in the Annotated Instructor Edition.

Both sets of notes are available in MyMathLab and can be customized by the instructor.

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Resources for Success

Student Supplements

Student's Solutions Manual

By Beverly Fusfield

- Provides detailed solutions to all odd-numbered text exercises

ISBN: 0-13-431021-7 & 978-0-13-431021-3

Video Lectures with Optional Captioning

- Feature Quick Reviews and Example Solutions:
  Quick Reviews cover key definitions and procedures from each section.
  Example Solutions walk students through the detailed solution process for every example in the textbook.
- Ideal for distance learning or supplemental instruction at home or on campus
- Include optional text captioning
- Available in MyMathLab®

MyNotes

- Available in MyMathLab and offer structure for students as they watch videos or read the text
- Include textbook examples along with ample space for students to write solutions and notes
- Include key concepts along with prompts for students to read, write, and reflect on what they have just learned
- **Customizable** so that instructors can add their own examples or remove examples that are not covered in their courses

MyClassroomExamples

- Available in MyMathLab and offer structure for classroom lecture
- Include Classroom Examples along with ample space for students to write solutions and notes
- Include key concepts along with fill in the blank opportunities to keep students engaged
- **Customizable** so that instructors can add their own examples or remove Classroom Examples that are not covered in their courses

Instructor Supplements

Annotated Instructor's Edition

- Provides answers in the margins to almost all text exercises, as well as helpful Teaching Tips and Classroom Examples
- Includes sample homework assignments indicated by exercise numbers underlined in blue within each end-of-section exercise set
- Sample homework exercises assignable in MyMathLab

ISBN: 0-13-421764-0 & 978-0-13-421764-2

Online Instructor's Solutions Manual

By Beverly Fusfield

- Provides complete solutions to all text exercises
- Available in MyMathLab or downloadable from Pearson Education's online catalog

Online Instructor's Testing Manual

By David Atwood

- Includes diagnostic pretests, chapter tests, final exams, and additional test items, grouped by section, with answers provided
- Available in MyMathLab or downloadable from Pearson Education's online catalog

TestGen®

- Enables instructors to build, edit, print, and administer tests
- Features a computerized bank of questions developed to cover all text objectives
- Available in MyMathLab or downloadable from Pearson Education's online catalog

Online PowerPoint Presentation and Classroom Example PowerPoints

- Written and designed specifically for this text
- Include figures and examples from the text
- Provide Classroom Example PowerPoints that include full worked-out solutions to all Classroom Examples
- Available in MyMathLab or downloadable from Pearson Education's online catalog

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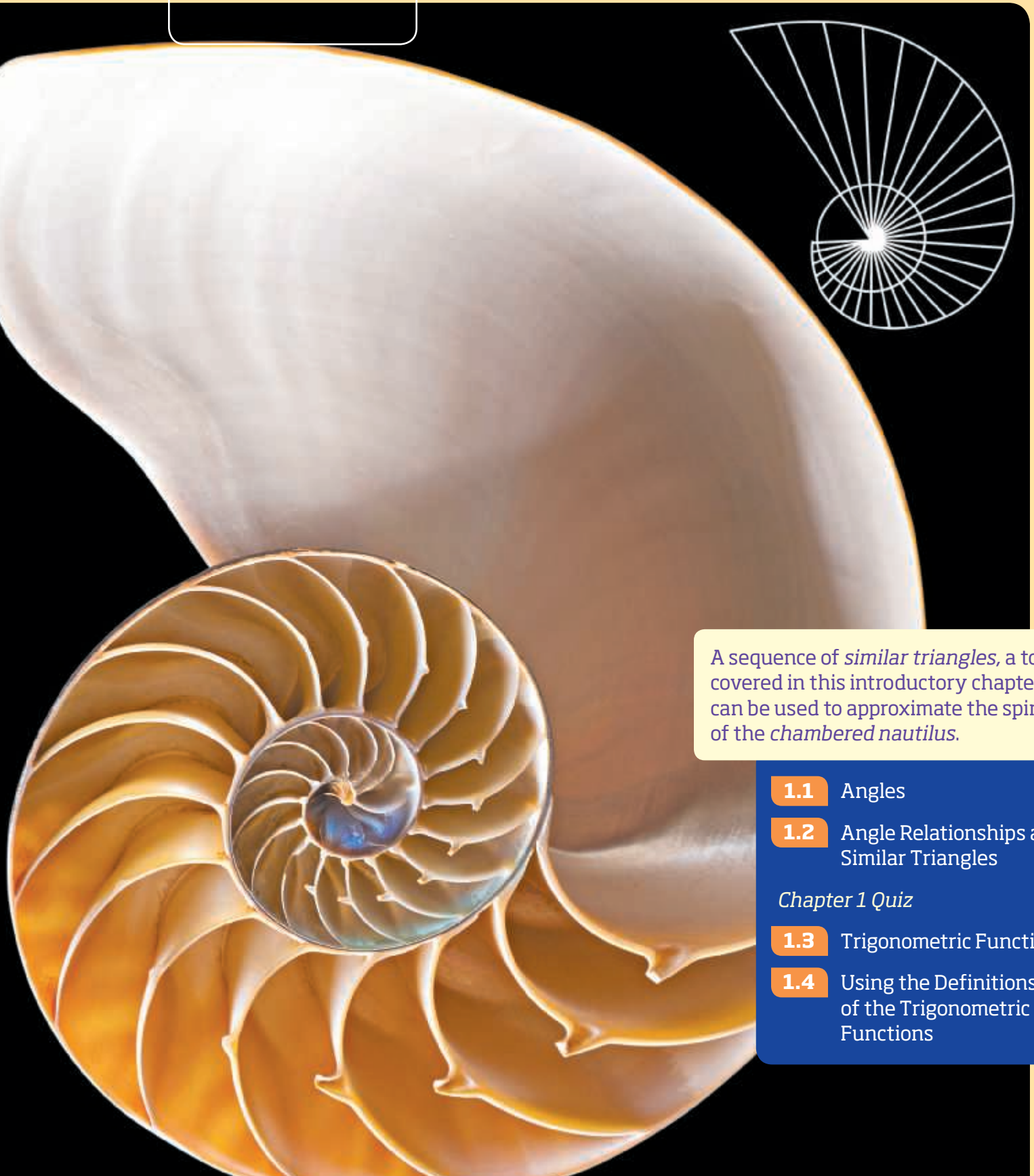
As an author team, we are committed to providing the best possible college algebra course to help instructors teach and students succeed. As we continue to work toward this goal, we welcome any comments or suggestions you might send, via e-mail, to math@pearson.com.

Margaret L. Lial
John Hornsby
David I. Schneider
Callie J. Daniels

1 Trigonometric Functions



A sequence of *similar triangles*, a topic covered in this introductory chapter, can be used to approximate the spiral of the *chambered nautilus*.

1.1 Angles

- Basic Terminology
- Degree Measure
- Standard Position
- Coterminal Angles

Basic Terminology

Two distinct points A and B determine a line called **line AB.** The portion of the line between A and B, including points A and B themselves, is **line segment AB,** or simply **segment AB.** The portion of line AB that starts at A and continues through B, and on past B, is the **ray AB.** Point A is the **endpoint of the ray.** See **Figure 1.**

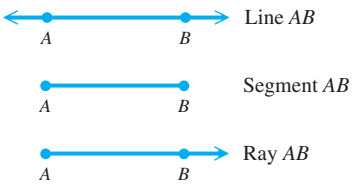


Figure 1

In trigonometry, an **angle** consists of two rays in a plane with a common endpoint, or two line segments with a common endpoint. These two rays (or segments) are the **sides** of the angle, and the common endpoint is the **vertex** of the angle. Associated with an angle is its measure, generated by a rotation about the vertex. See **Figure 2.** This measure is determined by rotating a ray starting at one side of the angle, the **initial side,** to the position of the other side, the **terminal side.** ***A counterclockwise rotation generates a positive measure, and a clockwise rotation generates a negative measure.*** The rotation can consist of more than one complete revolution.

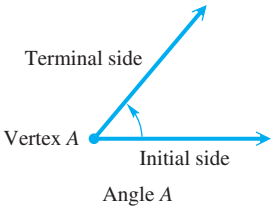


Figure 2

Figure 3 shows two angles, one **positive** and one **negative.**

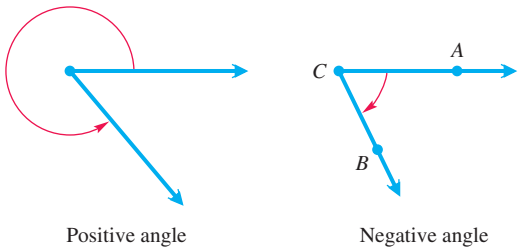
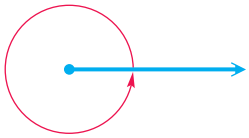


Figure 3

An angle can be named by using the name of its vertex. For example, the angle on the right in **Figure 3** can be named angle C. Alternatively, an angle can be named using three letters, with the vertex letter in the middle. Thus, the angle on the right also could be named angle ACB or angle BCA.

Degree Measure

The most common unit for measuring angles is the **degree.** Degree measure was developed by the Babylonians 4000 yr ago. To use degree measure, we assign 360 degrees to a complete rotation of a ray.* In **Figure 4,** notice that the terminal side of the angle corresponds to its initial side when it makes a complete rotation.



A complete rotation of a ray gives an angle whose measure is 360°. $\frac{1}{360}$ of a complete rotation gives an angle whose measure is 1°.

Figure 4

$$\text{One degree, written } 1^\circ\text{, represents } \frac{1}{360} \text{ of a complete rotation.}$$

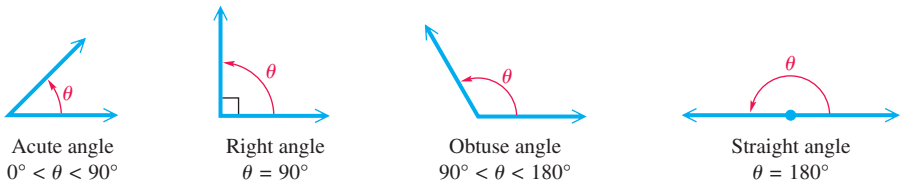
Therefore, 90° represents $\frac{90}{360} = \frac{1}{4}$ of a complete rotation, and 180° represents $\frac{180}{360} = \frac{1}{2}$ of a complete rotation.

An angle measuring between 0° and 90° is an **acute angle.** An angle measuring exactly 90° is a **right angle.** The symbol $\urcorner$ is often used at the vertex of a right angle to denote the 90° measure. An angle measuring more than 90° but less than 180° is an **obtuse angle,** and an angle of exactly 180° is a **straight angle.**

*The Babylonians were the first to subdivide the circumference of a circle into 360 parts. There are various theories about why the number 360 was chosen. One is that it is approximately the number of days in a year, and it has many divisors, which makes it convenient to work with in computations.

The Greek Letters		
A	α	alpha
B	β	beta
Γ	γ	gamma
Δ	δ	delta
E	ε	epsilon
Z	ζ	zeta
H	η	eta
Θ	θ	theta
I	ι	iota
K	κ	kappa
Λ	λ	lambda
M	μ	mu
N	ν	nu
Ξ	ξ	xi
O	o	omicron
Π	π	pi
P	ρ	rho
Σ	σ	sigma
T	τ	tau
Υ	υ	upsilon
Φ	ϕ	phi
X	χ	chi
Ψ	ψ	psi
Ω	ω	omega

In **Figure 5,** we use the **Greek letter θ (theta)*** to name each angle. The table in the margin lists the upper- and lowercase Greek letters, which are often used in trigonometry.



Acute angle $0° < \theta < 90°$ | Right angle $\theta = 90°$ | Obtuse angle $90° < \theta < 180°$ | Straight angle $\theta = 180°$

Figure 5

If the sum of the measures of two positive angles is 90°, the angles are **complementary** and the angles are **complements** of each other. Two positive angles with measures whose sum is 180° are **supplementary,** and the angles are **supplements.**

EXAMPLE 1 Finding the Complement and the Supplement of an Angle

Find the measure of **(a)** the complement and **(b)** the supplement of an angle measuring 40°.

SOLUTION

(a) To find the measure of its complement, subtract the measure of the angle from 90°.

$$90° - 40° = 50° \quad \text{Complement of } 40°$$

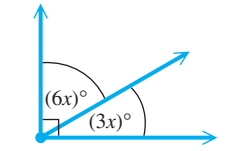
(b) To find the measure of its supplement, subtract the measure of the angle from 180°.

$$180° - 40° = 140° \quad \text{Supplement of } 40°$$

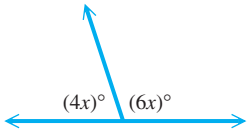
✔ **Now Try Exercise 11.**

EXAMPLE 2 Finding Measures of Complementary and Supplementary Angles

Find the measure of each marked angle in **Figure 6.**



(a)



(b)

Figure 6

SOLUTION

(a) Because the two angles in **Figure 6(a)** form a right angle, they are complementary angles.

$$6x + 3x = 90 \quad \text{Complementary angles sum to } 90°.$$

$$9x = 90 \quad \text{Combine like terms.}$$

Don't stop here.

$$x = 10 \quad \text{Divide by } 9.$$

Be sure to determine the measure of each angle by substituting 10 for x in $6x$ and $3x$. The two angles have measures of $6(10) = 60°$ and $3(10) = 30°$.

(b) The angles in **Figure 6(b)** are supplementary, so their sum must be 180°.

$$4x + 6x = 180 \quad \text{Supplementary angles sum to } 180°.$$

$$10x = 180 \quad \text{Combine like terms.}$$

$$x = 18 \quad \text{Divide by } 10.$$

The angle measures are $4x = 4(18) = 72°$ and $6x = 6(18) = 108°$.

✔ **Now Try Exercises 23 and 25.**

* In addition to θ (theta), other Greek letters such as α (alpha) and β (beta) are used to name angles.

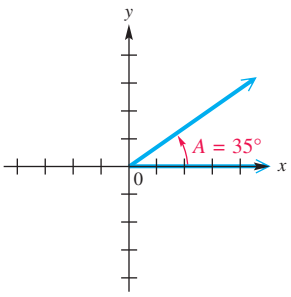


Figure 7

The measure of angle A in **Figure 7** is 35°. This measure is often expressed by saying that $\boldsymbol{m(\text{angle } A)}$ is 35°, where $m(\text{angle } A)$ is read **"the measure of angle A."** The symbolism $m(\text{angle } A) = 35°$ is abbreviated as $A = 35°$.

Traditionally, portions of a degree have been measured with minutes and seconds. One **minute,** written $\mathbf{1'}$, is $\frac{1}{60}$ of a degree.

$$\mathbf{1' = \frac{1}{60}^\circ} \quad \text{or} \quad \mathbf{60' = 1^\circ}$$

One **second,** $\mathbf{1''}$, is $\frac{1}{60}$ of a minute.

$$\mathbf{1'' = \frac{1}{60}' = \frac{1}{3600}^\circ} \quad \text{or} \quad \mathbf{60'' = 1'} \text{ and } \mathbf{3600'' = 1^\circ}$$

The measure $12° 42' 38''$ represents 12 degrees, 42 minutes, 38 seconds.

EXAMPLE 3 Calculating with Degrees, Minutes, and Seconds

Perform each calculation.

(a) $51° 29' + 32° 46'$ **(b)** $90° - 73° 12'$

SOLUTION

(a)
$$\begin{array}{r} 51° 29' \\ +\ 32° 46' \\ \hline 83° 75' \end{array}$$
Add degrees and minutes separately.

The sum $83° 75'$ can be rewritten as follows.

$$\begin{aligned} & 83° 75' \\ &= 83° + 1° 15' && 75' = 60' + 15' = 1° 15' \\ &= 84° 15' && \text{Add.} \end{aligned}$$

(b)
$$\begin{array}{r} 90° \\ -\ 73° 12' \\ \hline \end{array} \quad \text{can be written} \quad \begin{array}{r} 89° 60' \\ -\ 73° 12' \\ \hline 16° 48' \end{array}$$
Write $90°$ as $89° 60'$.

Now Try Exercises 41 and 45.

An alternative way to measure angles involves decimal degrees. For example,

$$12.4238° \quad \text{represents} \quad 12\frac{4238}{10{,}000}^\circ.$$

EXAMPLE 4 Converting between Angle Measures

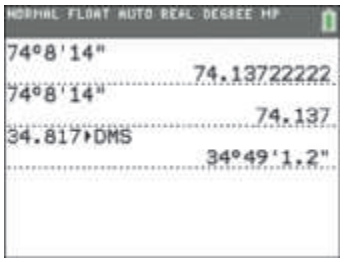
(a) Convert $74° 08' 14''$ to decimal degrees to the nearest thousandth.

(b) Convert $34.817°$ to degrees, minutes, and seconds to the nearest second.

SOLUTION

(a) $74° 08' 14''$

$$\begin{aligned} &= 74° + \frac{8}{60}^\circ + \frac{14}{3600}^\circ && 08' \cdot \tfrac{1°}{60'} = \tfrac{8}{60}^\circ \text{ and } 14'' \cdot \tfrac{1°}{3600''} = \tfrac{14}{3600}^\circ \\ &\approx 74° + 0.1333° + 0.0039° && \text{Divide to express the fractions as decimals.} \\ &\approx 74.137° && \text{Add and round to the nearest thousandth.} \end{aligned}$$



This screen shows how the TI-84 Plus performs the conversions in **Example 4.** The ▶DMS option is found in the ANGLE Menu.

(b) $34.817°$

$$= 34° + 0.817° \qquad \text{Write as a sum.}$$
$$= 34° + 0.817(60') \qquad 0.817° \cdot \frac{60'}{1°} = 0.817(60')$$
$$= 34° + 49.02' \qquad \text{Multiply.}$$
$$= 34° + 49' + 0.02' \qquad \text{Write } 49.02' \text{ as a sum.}$$
$$= 34° + 49' + 0.02(60'') \qquad 0.02' \cdot \frac{60''}{1'} = 0.02(60'')$$
$$= 34° + 49' + 1.2'' \qquad \text{Multiply.}$$
$$\approx 34°\,49'\,01'' \qquad \text{Approximate to the nearest second.}$$

Now Try Exercises 61 and 71.

Standard Position An angle is in **standard position** if its vertex is at the origin and its initial side lies on the positive x-axis. The angles in **Figures 8(a) and 8(b)** are in standard position. An angle in standard position is said to lie in the quadrant in which its terminal side lies. An acute angle is in quadrant I (**Figure 8(a)**) and an obtuse angle is in quadrant II (**Figure 8(b)**). **Figure 8(c)** shows ranges of angle measures for each quadrant when $0° < \theta < 360°$.

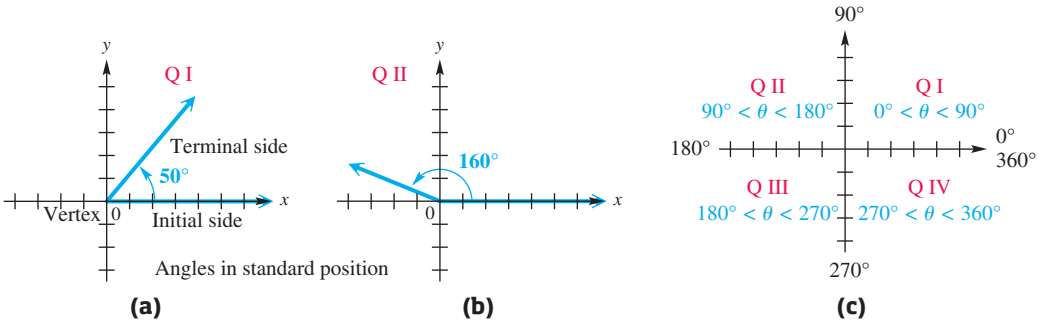


Figure 8

Quadrantal Angles

Angles in standard position whose terminal sides lie on the x-axis or y-axis, such as angles with measures 90°, 180°, 270°, and so on, are **quadrantal angles.**

Coterminal Angles A complete rotation of a ray results in an angle measuring 360°. By continuing the rotation, angles of measure larger than 360° can be produced. The angles in **Figure 9** with measures 60° and 420° have the same initial side and the same terminal side, but different amounts of rotation. Such angles are **coterminal angles.** ***Their measures differ by a multiple of* 360°.** As shown in **Figure 10,** angles with measures 110° and 830° are coterminal.

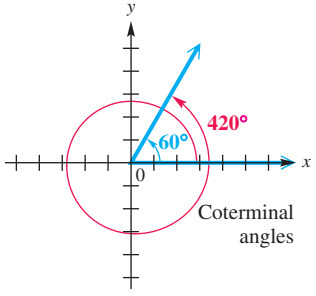


Figure 9

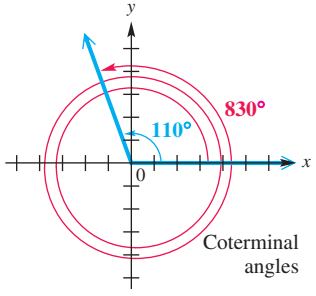


Figure 10

EXAMPLE 5 **Finding Measures of Coterminal Angles**

Find the angle of least positive measure that is coterminal with each angle.

(a) 908° **(b)** −75° **(c)** −800°

SOLUTION

(a) Subtract 360° as many times as needed to obtain an angle with measure greater than 0° but less than 360°.

$$908° - 2 \cdot 360° = 188° \quad \text{Multiply } 2 \cdot 360°\text{. Then subtract.}$$

An angle of 188° is coterminal with an angle of 908°. See **Figure 11.**

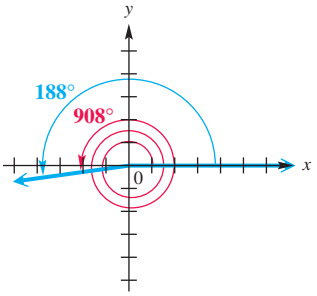


Figure 11

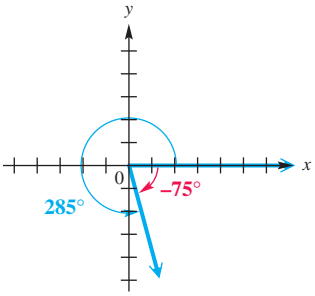


Figure 12

(b) Add 360° to the given negative angle measure to obtain the angle of least *positive* measure. See **Figure 12.**

$$-75° + 360° = 285°$$

(c) The least integer multiple of 360° greater than 800° is

$$3 \cdot 360° = 1080°.$$

Add 1080° to −800° to obtain

$$-800° + 1080° = 280°.$$

Now Try Exercises 81, 91, and 95.

Sometimes it is necessary to find an expression that will generate all angles coterminal with a given angle. For example, we can obtain any angle coterminal with 60° by adding an integer multiple of 360° to 60°. Let n represent any integer. Then the following expression represents all such coterminal angles.

$$60° + \boldsymbol{n} \cdot \mathbf{360°} \quad \text{Angles coterminal with } 60°$$

The table below shows a few possibilities.

Examples of Angles Coterminal with 60°

Value of n	Angle Coterminal with 60°
2	$60° + 2 \cdot 360° = 780°$
1	$60° + 1 \cdot 360° = 420°$
0	$60° + 0 \cdot 360° = 60°$ (the angle itself)
−1	$60° + (-1) \cdot 360° = -300°$
−2	$60° + (-2) \cdot 360° = -660°$

This table shows some examples of coterminal quadrantal angles.

Examples of Coterminal Quadrantal Angles

Quadrantal Angle θ	Coterminal with θ
$0°$	$\pm 360°, \pm 720°$
$90°$	$-630°, -270°, 450°$
$180°$	$-180°, 540°, 900°$
$270°$	$-450°, -90°, 630°$

EXAMPLE 6 **Analyzing Revolutions of a Disk Drive**

A constant angular velocity disk drive spins a disk at a constant speed. Suppose a disk makes 480 revolutions per min. Through how many degrees will a point on the edge of the disk move in 2 sec?

SOLUTION The disk revolves 480 times in 1 min, or $\frac{480}{60}$ times $= 8$ times per sec (because 60 sec $=$ 1 min). In 2 sec, the disk will revolve $2 \cdot 8 = 16$ times. Each revolution is 360°, so in 2 sec a point on the edge of the disk will revolve

$$16 \cdot 360° = 5760°.$$

A unit analysis expression can also be used.

$$\frac{480 \text{ rev}}{1 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{360°}{1 \text{ rev}} \times 2 \text{ sec} = 5760° \quad \text{Divide out common units.}$$

Now Try Exercise 123.

1.1 Exercises

CONCEPT PREVIEW *Fill in the blank(s) to correctly complete each sentence.*

1. One degree, written 1°, represents _____ of a complete rotation.
2. If the measure of an angle is $x°$, its complement can be expressed as _____ $- x°$.
3. If the measure of an angle is $x°$, its supplement can be expressed as _____ $- x°$.
4. The measure of an angle that is its own complement is _____.
5. The measure of an angle that is its own supplement is _____.
6. One minute, written $1'$, is _____ of a degree.
7. One second, written $1''$, is _____ of a minute.
8. $12° 30'$ written in decimal degrees is _____.
9. 55.25° written in degrees and minutes is _____.
10. If n represents any integer, then an expression representing all angles coterminal with 45° is $45° +$ _____.

Find the measure of **(a)** *the complement and* **(b)** *the supplement of an angle with the given measure.* ***See Examples 1 and 3.***

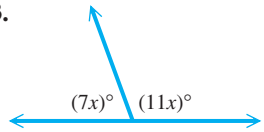
11. 30° **12.** 60° **13.** 45° **14.** 90°

15. 54° **16.** 10° **17.** 1° **18.** 89°

19. $14° 20'$ **20.** $39° 50'$ **21.** $20° 10' 30''$ **22.** $50° 40' 50''$

Find the measure of each marked angle. ***See Example 2.***

23.



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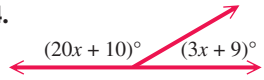

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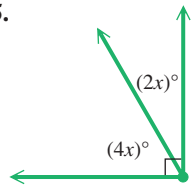

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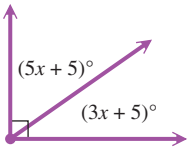

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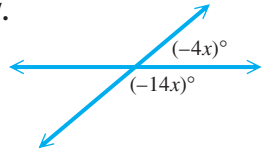

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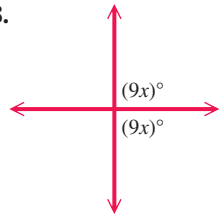


29. supplementary angles with measures $10x + 7$ and $7x + 3$ degrees

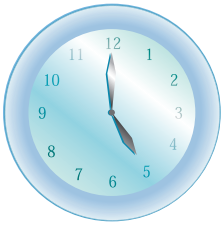
30. supplementary angles with measures $6x - 4$ and $8x - 12$ degrees

31. complementary angles with measures $9x + 6$ and $3x$ degrees

32. complementary angles with measures $3x - 5$ and $6x - 40$ degrees

Find the measure of the smaller angle formed by the hands of a clock at the following times.

33.



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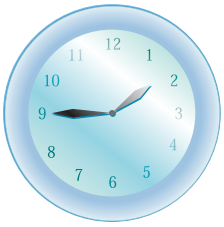


35. 3:15 **36.** 9:45 **37.** 8:20 **38.** 6:10

Perform each calculation. ***See Example 3.***

39. $62° 18' + 21° 41'$ **40.** $75° 15' + 83° 32'$ **41.** $97° 42' + 81° 37'$

42. $110° 25' + 32° 55'$ **43.** $47° 29' - 71° 18'$ **44.** $47° 23' - 73° 48'$

45. $90° - 51° 28'$ **46.** $90° - 17° 13'$ **47.** $180° - 119° 26'$

48. $180° - 124° 51'$ **49.** $90° - 72° 58' 11''$ **50.** $90° - 36° 18' 47''$

51. $26° 20' + 18° 17' - 14° 10'$ **52.** $55° 30' + 12° 44' - 8° 15'$

Convert each angle measure to decimal degrees. If applicable, round to the nearest thousandth of a degree. ***See Example 4(a).***

53. $35° 30'$ **54.** $82° 30'$ **55.** $112° 15'$ **56.** $133° 45'$

57. $-60° 12'$ **58.** $-70° 48'$ **59.** $20° 54' 36''$ **60.** $38° 42' 18''$

61. $91° 35' 54''$ **62.** $34° 51' 35''$ **63.** $274° 18' 59''$ **64.** $165° 51' 09''$

Convert each angle measure to degrees, minutes, and seconds. If applicable, round to the nearest second. ***See Example 4(b).***

65. $39.25°$ **66.** $46.75°$ **67.** $126.76°$ **68.** $174.255°$

69. $-18.515°$ **70.** $-25.485°$ **71.** $31.4296°$ **72.** $59.0854°$

73. $89.9004°$ **74.** $102.3771°$ **75.** $178.5994°$ **76.** $122.6853°$

Find the angle of least positive measure (not equal to the given measure) that is coterminal with each angle. ***See Example 5.***

77. 32°	**78.** 86°	**79.** 26° 30′	**80.** 58° 40′
81. −40°	**82.** −98°	**83.** −125° 30′	**84.** −203° 20′
85. 361°	**86.** 541°	**87.** −361°	**88.** −541°
89. 539°	**90.** 699°	**91.** 850°	**92.** 1000°
93. 5280°	**94.** 8440°	**95.** −5280°	**96.** −8440°

Give two positive and two negative angles that are coterminal with the given quadrantal angle.

97. 90°	**98.** 180°	**99.** 0°	**100.** 270°

Write an expression that generates all angles coterminal with each angle. Let n represent any integer.

101. 30°	**102.** 45°	**103.** 135°	**104.** 225°
105. −90°	**106.** −180°	**107.** 0°	**108.** 360°

109. Why do the answers to **Exercises 107 and 108** give the same set of angles?

110. ***Concept Check*** Which two of the following are not coterminal with $r°$?

A. $360° + r°$ **B.** $r° - 360°$ **C.** $360° - r°$ **D.** $r° + 180°$

Concept Check *Sketch each angle in standard position. Draw an arrow representing the correct amount of rotation. Find the measure of two other angles, one positive and one negative, that are coterminal with the given angle. Give the quadrant of each angle, if applicable.*

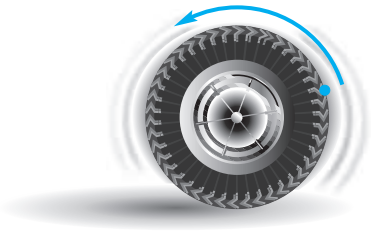
111. 75°	**112.** 89°	**113.** 174°	**114.** 234°
115. 300°	**116.** 512°	**117.** −61°	**118.** −159°
119. 90°	**120.** 180°	**121.** −90°	**122.** −180°

Solve each problem. ***See Example 6.***

123. ***Revolutions of a Turntable*** A turntable in a shop makes 45 revolutions per min. How many revolutions does it make per second?

124. ***Revolutions of a Windmill*** A windmill makes 90 revolutions per min. How many revolutions does it make per second?

125. ***Rotating Tire*** A tire is rotating 600 times per min. Through how many degrees does a point on the edge of the tire move in $\frac{1}{2}$ sec?



126. ***Rotating Airplane Propeller*** An airplane propeller rotates 1000 times per min. Find the number of degrees that a point on the edge of the propeller will rotate in 2 sec.

127. ***Rotating Pulley*** A pulley rotates through 75° in 1 min. How many rotations does the pulley make in 1 hr?